EXAMPLE

Aluminum fins 1.5 cm wide and 1.0 mm thick are placed on a 2.5-cm-diameter tube to dissipate the heat. The tube surface temperature is 170◦, and the ambient-fluid temperature is 25◦C. Calculate the heat loss per fin for h = 130 W/m2 · ◦C. Assume k = 200 W/m · ◦C for aluminum.

Solution:

For this example we can compute the heat transfer by using the fin-efficiency curves in Figure 2-18. The parameters needed are

$$L_c = L + \frac{t}{2} = 1.5 + 0.05 = 1.55\ cm$$

$$r_1 = \frac{2.5}{2} = 1.25\ cm$$

$$r_{2c} = r_1 + L_c = 1.25 + 1.55 = 2.80\ cm$$

$$\frac{r_{2c}}{r_1} = \frac{2.80}{1.25} = 2.24$$

$A_m = tL_c = (0.001)(1.55 * 10^{-2}) = 1.55 * 10^{-5}\ m^2$, Profile area

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0155)^{3/2}\left(\frac{130}{(200)(1.5*10^{-5})}\right)^{1/2} = 0.396$$

From Figure 2-18, η_f = 82 percent. The heat that would be transferred if the entire fin were at the base temperature is (both sides of fin exchanging heat)

$$q_{f,max} = 2\pi(r_{2c}^2 - r_1^2)h(T_b - T_\infty)$$

$$q_{f,max} = 2\pi(2.8^2 - 1.25^2)(10^{-4})(130)(170 - 25)$$

$$q_{f,max} = 74.35\ \text{W.}$$

$$q_{f,act} = \eta_f q_{f,max} = (0.82)(74.35) = 60.97\ \text{W}$$

*Fin Effectiveness

Fins are used to enhance heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that adding fins on a surface will enhance heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the fin effectiveness ε_f, defined as, Figure 2.19.

$$\varepsilon_f = \frac{q_f}{q_{no,f}}$$

$$= \frac{q_f}{hA_b(T_b - T_\infty)}$$

$$= \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface of area } A_b}$$

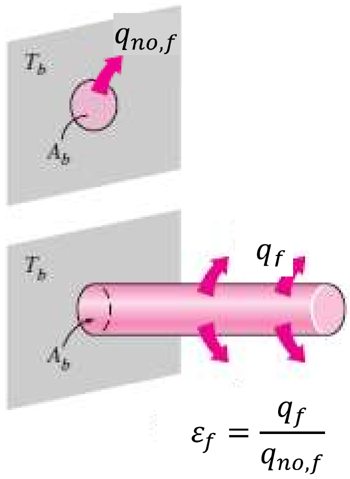


Figure 2.19: The effectivness of the fin.

A_b : cross sectional area of the fin, m^2.

ε_f : fin effectivness.

q_f : heat transfer rate from the fin, m^2 .

$q_{no,f}$: heat transfer rate from the base area without fin, m^2.

$\varepsilon_f = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all.

$\varepsilon_f < 1$ indicates that the fin actually acts as insulation, slowing down the heat transfer from the surface.

$\varepsilon_f > 1$ indicates that fins are enhancing heat transfer from the surface, as they should.

However, the use of fins cannot be justified unless fin is sufficiently larger than 1. Finned surfaces are designed on the basis of maximizing effectiveness for a specified cost or minimizing cost for a desired effectiveness. Note that both the fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_f = \frac{q_f}{q_{no,f}}$$

$$= \frac{q_f}{hA_b(T_b - T_\infty)}$$

$$= \frac{\eta_f hA_f(T_b - T_\infty)}{hA_b(T_b - T_\infty)}$$

$$\varepsilon_f = \frac{A_f}{A_b}\eta_f$$

Case A : For uniform cross section and long fin Case A, the effectivness is given as:

$$\varepsilon_f = \frac{\sqrt{hPkA_c}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}\theta_b}{hA_b\theta_b}$$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

Where $A_b = A_c$

Case B: For insulated tip fin:

$$\varepsilon_f = \frac{\sqrt{hPkA_c}\theta_b \tanh mL}{hA_b\theta_b}$$

$$\varepsilon_f = \frac{\tanh mL}{\sqrt{\frac{hA_c}{kP}}}$$

or

$$\varepsilon_f = \frac{A_f}{A_b}\eta_f$$

$$= \frac{PL}{A_c}\frac{\tanh mL}{mL}$$

Because $m^2 = \frac{hP}{kA_c}$

Therefore:

$$\varepsilon_f = \frac{P}{A_c}\frac{\tanh mL}{\sqrt{\frac{hP}{kA_c}}}$$

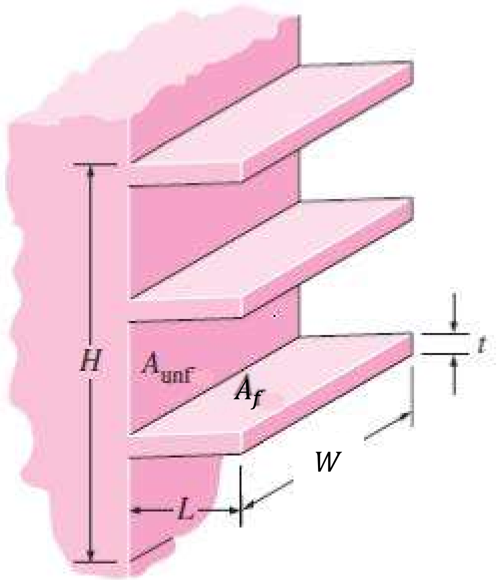
$$\varepsilon_f = \frac{\tanh mL}{\sqrt{\frac{hA_c}{kP}}}$$

When determining the rate of heat transfer from a finned surface, we must consider the *un-finned portion* of the surface as well as the *fins*. Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$q_{total,f} = q_{un,f} + q_f$$

$$q_{total,f} = hA_{unf}(T_b - T_\infty) + \eta_f hA_f(T_b - T_\infty)$$

$$q_{total,f} = h(A_{un,f} + \eta_f A_f)(T_b - T_\infty)$$



$$A_{no,f} = WH$$

$$A_{un,f} = WH - 3tW$$

$$A_f = 2LW + tW$$

$A_f \cong 2LW$

Example:

Steam in a heating system flows through tubes whose outer diameter is D_1= 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k =180 W/m · °C) of outer diameter D_2 =6 cm and constant thickness t= 2 mm are attached to the tube, as shown in Figure below. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞= 25°C, with a combined heat transfer coefficient of h = 60 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

Solution:

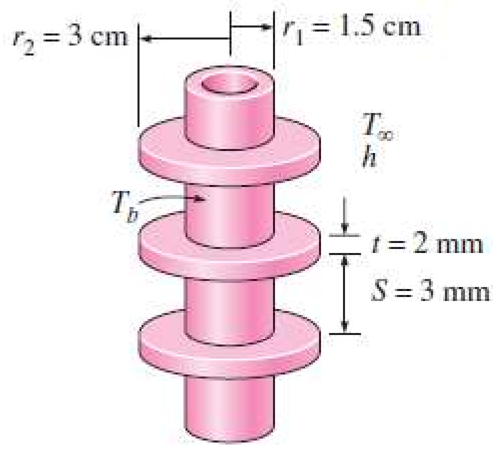
In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$A_{no,f} = \pi D_1 L = \pi(0.03)(1) = 0.0942 \text{ m}^2$

$q_{no,f} = hA_{no,f}(T_b - T_\infty)$

$q_{no,f} = (60)(0.0942)(120 - 25)$

$q_{no,f} = 537 \text{ W}$



$L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m

$L_c = L + \frac{t}{2} = 0.015 + 0.001 = 0.016$ m

$r_{2c} = r_1 + L_c = 0.015 + 0.016 = 0.031$ m

$\frac{r_{2c}}{r_1} = \frac{0.031}{0.015} = 2.067$

$A_m = tL_c = (0.002)(0.016) = 3.2 * 10^{-5}$ m^2, Profile area

$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.016)^{3/2}\left(\frac{60}{(180)(3.2*10^{-5})}\right)^{1/2} = 0.206$

From figure 2.18

$\eta_f = 0.95$

$A_f = 2\pi(r_{2c}^2 - r_1^2)$

$A_f = 2\pi(0.03^2 - 0.015^2) = 0.00462$ m^2

$q_f = \eta_f q_{f,max} = \eta_f hA_f(T_b - T_\infty)$

$q_f = 0.95(60)(0.00462)(120 - 25) = 25$ W

$A_{un,f} = \pi D_1 S = \pi(0.03)(0.003) = 0.000283$ m^2

$q_{un,f} = hA_{un,f}(T_b - T_\infty)$

$q_{un,f} = (60)(0.000283)(120 - 25) = 1.6$ W

Noting that there are 200 fins and thus 200 inter-fin spacing's per meter length of the tube, the total heat transfer from the finned tube becomes

$q_{total,f} = n(q_{un,f} + q_f) = 200(25 + 1.6) = 5320$ W

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$q_{increase} = q_{total,f} - q_{no,f} = 5320 - 537 = 4783$ W (per m tube length)

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320 \text{ W}}{537 \text{ W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

Example:

Steam in a heating system flows through tubes whose outer diameter is D_1= 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k =180 W/m · °C) of outer diameter D_2 =6 cm and constant thickness t= 2 mm are attached to the tube, as shown in Figure below. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞= 25°C, with a combined heat transfer coefficient of h = 60 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

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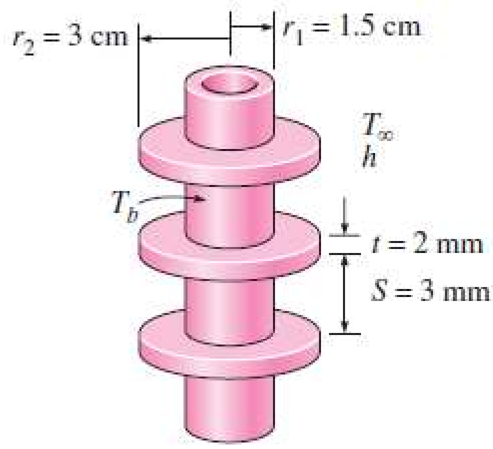
In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$A_{no,f} = \pi D_1 L = \pi(0.03)(1) = 0.0942$ m^2

$q_{no,f} = hA_{no,f}(T_b - T_\infty)$

$q_{no,f} = (60)(0.0942)(120 - 25)$

$q_{no,f} = 537$ W



$L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m

$L_c = L + \frac{t}{2} = 0.015 + 0.001 = 0.016$ m

$r_{2c} = r_1 + L_c = 0.015 + 0.016 = 0.031$ m

$\frac{r_{2c}}{r_1} = \frac{0.031}{0.015} = 2.067$

$A_m = tL_c = (0.002)(0.016) = 3.2 * 10^{-5}$ m^2, Profile area

$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.016)^{3/2}\left(\frac{60}{(180)(3.2*10^{-5})}\right)^{1/2} = 0.206$

From figure 2.18

$\eta_f = 0.95$

$A_f = 2\pi(r_{2c}^2 - r_1^2)$

$A_f = 2\pi(0.03^2 - 0.015^2) = 0.00462$ m^2

$q_f = \eta_f q_{f,max} = \eta_f h A_f (T_b - T_\infty)$

$q_f = 0.95(60)(0.00462)(120 - 25) = 25$ W

$A_{un,f} = \pi D_1 S = \pi(0.03)(0.003) = 0.000283$ m^2

$q_{un,f} = hA_{un,f}(T_b - T_\infty)$

$q_{un,f} = (60)(0.000283)(120 - 25) = 1.6$ W

Noting that there are 200 fins and thus 200 inter fin spacing per meter length of the tube, the total heat transfer from the finned tube becomes

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That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

Holman book

2.33: A circumferential fin of rectangular profile is constructed of stainless steel with k = 43 W/m · °C and a thickness of 1.0 mm. The fin is installed on a tube having a diameter of 3.0 cm and the outer radius of the fin is 4.0 cm. The inner tube is maintained at 250°C and the assembly is exposed to a convection environment having $T\infty$ =35°C and h = 45 W/m^2 . °C. Calculate the heat lost by the fin.

Solution:

$L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.08 - 0.03) = 0.025$

$L_c = L + \frac{t}{2} = 0.025 + \frac{0.001}{2} = 0.0255\, m$

$r_{2c} = r_1 + L_c = 0.015 + 0.0255 = 0.0405$

$\frac{r_{2c}}{r_1} = \frac{0.0405}{0.015} = 2.7$

$A_m = tL_c = (0.001)(0.0255) = 2.55 * 10^{-5}$ m^2

$L_c{}^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0255)^{3/2}\left(\frac{45}{(43)(2.55*10^{-5})}\right)^{1/2} = 0.825$

From figure 2.18 $\eta_f = 0.59$

$q_{f,max} = 2\pi(r_{2c}^2 - r_1^2)h(T_b - T_\infty)$

$q_{f,max} = (43)2\pi((0.0405)^2 - (0.015)^2)(250 - 35) = 82.2$ W

$q_{f,act} = \eta_f q_{f,max} = (0.59)(82.2) = 48.5$ W

2.63: A circumferential fin of rectangular profile has a thickness of 0.7mm and is installed on a tube having a diameter of 3 cm that is maintained at a temperature of 200◦C. The length of the fin is 2 cm and the fin material is copper. Calculate the heat lost by the fin to a surrounding convection environment at 100◦C with a convection heat-transfer coefficient of 524 W/m2 · ◦C.

2-74 A straight fin of rectangular profile has a thermal conductivity of 14 W/m · °C, thickness of 2.0 mm, and length of 23 mm. The base of the fin is maintained at a temperature of 220°C while the fin is exposed to a convection environment at 23°C with $h = 25$ W/m^2 · °C. Calculate the heat lost per meter of fin depth.

Solution:

$$L_c = 23 + 1 = 24 \text{ mm}$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.024)^{3/2}\left[\frac{25}{(14)(0.002)(0.024)}\right]^{1/2} = 0.717$$

From figure 2.17 rectangular profile area $\eta_f = 0.77$

$$q_{f,max} = 2(L_c w)h(T_b - T_\infty)$$

$$q_{f,max}/w = 2(0.024)(25)(220 - 23) = 236.36 \text{ W/m}$$

$$q_{f,act} = \eta_f q_{f,max} = (0.77)(236.36) = 182 \text{ W/m}$$

2-76 The total efficiency for a finned surface may be defined as the ratio of the total heat transfer of the combined area of the surface and fins to the heat that would be transferred if this total area were maintained at the base temperature T_0. Show that this efficiency can be calculated from

$$\eta_t = 1 - \frac{A_f}{A}(1 - \eta_f)$$

where

η_t = total efficiency
A_f = surface area of all fins
A = total heat-transfer area, including fins and exposed tube or other surface
η_f = fin efficiency

$$q_{max} = hA(T_b - T_\infty)$$

$$\eta_t = \frac{q_{act}}{q_{max}}$$

$$\eta_t = \frac{h(A - A_f)(T_b - T_\infty) + \eta_f A_f h(T_b - T_\infty)}{hA(T_b - T_\infty)}$$

$\eta_t = \frac{(A-A_f)+\eta_f A_f}{A}$

$\eta_t = 1 - \frac{A_f}{A} + \frac{A_f}{A}\eta_f$

$\eta_t = 1 - \frac{A_f}{A}(1-\eta_f)$

2-75 A circumferential fin of rectangular profile is constructed of a material having $k = 55$ W/m · °C and is installed on a tube having a diameter of 3 cm. The length of fin is 3 cm and the thickness is 2 mm. If the fin is exposed to a convection environment at 20°C with a convection coefficient of 68 W/m^2 · °C and the tube wall temperature is 100°C, calculate the heat lost by the fin.

2-77: A triangular fin of stainless steel (18% Cr, 8% Ni) is attached to a plane wall maintained at 460◦C. The fin thickness is 6.4 mm, and the length is 2.5 cm. The environment is at 93◦C, and the convection heat-transfer coefficient is 28 W/m2 · ◦C. Calculate the heat lost from the fin.

Solution:

T_b=460 ◦C t=6.4 mm L_c =L=2.5 cm T_∞=93 ◦C h=28 W/m^2 · ◦C k= 16.3 W/m .◦C

$$A_m = L\left(\frac{t}{2}\right) = 8\times10^{-5}\ \text{m}^2 \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.579$$

From figure 2.17 η_f=0.85

$$q_{f,max} = 2\left(\sqrt{\left(\frac{t}{2}\right)^2 + (L_c)^2} * w\right) h(T_b - T_\infty)$$

$$= 2\left(\sqrt{\left(\frac{0.0032}{2}\right)^2 + (0.025)^2} * 1\right)(28)(460-93) = 514.326\ \text{W}$$

$q_{f,act} = \eta_f q_{f,max} = (0.85)(514.326) = 437.17$

2-80 A straight rectangular fin 2.0 cm thick and 14 cm long is constructed of steel and placed on the outside of a wall maintained at 200◦C. The environment temperature is 15◦C, and the heat-transfer coefficient for convection is 20 W/m2 · ◦C. Calculate the heat lost from the fin per unit depth.

2-81 An aluminum fin 1.6 mm thick surrounds a tube 2.5 cm in diameter. The length of the fin is 12.5 mm. The tube-wall temperature is 200◦C, and the environment temperature is 20◦C. The heat-transfer coefficient is 60W/m2 · ◦C. What is the heat lost by the fin?

2-84 A circumferential fin of rectangular profile is installed on a 10-cm-diameter tube maintained at 120◦C. The fin has a length of 15 cm and thickness of 2 mm. The fin is exposed to a convection environment at 23◦C with *h* = 60 W/m2 · ◦C and the fin conductivity is 120 W/m · ◦C. Calculate the heat lost by the fin expressed in watts.

2-85 A long stainless-steel rod [*k* =16 W/m · ◦C] has a square cross section 12.5 by 12.5 mm and has one end maintained at 250◦C. The heat-transfer coefficient is 40 W/m2 · ◦C, and the environment temperature is 90◦C. Calculate the heat lost by the rod.

2-86 A straight fin of rectangular profile is constructed of duralumin (94% Al, 3% Cu) with a thickness of 2.1 mm. The fin is 17 mm long, and it is subjected to a convection environment with *h*=75 W/m2 · ◦C. If the base temperature is 100◦C and the environment is at 30◦C, calculate the heat transfer per unit length of fin.

2-87 A certain internal-combustion engine is air-cooled and has a cylinder constructed of cast iron [*k* =35 Btu/h · ft · ◦F]. The fins on the cylinder have a length of 58 in and thickness of 18 in. The convection coefficient is 12 Btu/h · ft2 · ◦F. The cylinder diameter is 4 in. Calculate the heat loss per fin for a base temperature of 450◦F and environment temperature of 100◦F.

2-88 A 1.5-mm-diameter stainless-steel rod [*k* =19 W/m · ◦C] protrudes from a wall maintained at 45◦C. The rod is 12 mm long, and the convection coefficient is 500W/m2 · ◦C. The environment temperature is 20◦C. Calculate the temperature of the tip of the rod. Repeat the calculation for *h*=200 and 1500 W/m2 · ◦C.

Solution:

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi \frac{D^2}{4}}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(500)}{19(0.0015)}} = 264.9$$

$$\frac{\theta(x)}{\theta_b} = \frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh m\,(L-x)+\frac{h}{km}\sinh m\,(L-x)}{\cosh mL+\frac{h}{km}\sinh mL}$$

At x=L

$$\frac{T(L)-T_\infty}{T_b-T_\infty} = \frac{\cosh m\,(L-L)+\frac{h}{km}\sinh m\,(L-L)}{\cosh mL+\frac{h}{km}\sinh mL} = \frac{1}{\cosh(264.9*0.012)+\frac{500}{(19*264.9)}\sinh(264.9*0.012)} =$$

0.0756 ◦C